Econ Theory
DOI 10.1007/s00199-010-0560-z

INTRODUCTION

Symposium: on the role of market belief in economic dynamics, an introduction

Mordecai Kurz

Papers in this symposium were presented at a Stanford University conference on August 10–14, 2009 entitled "When Are Diverse Beliefs Central?" They reflect the rapidly growing literature on the impact of diverse beliefs. It may thus be helpful to the reader if I use this introduction to highlight some ideas of this literature as reflected in the papers of this volume.

As the era of Rational Expectations (in short RE) comes to a close it is important to clarify two points. First, the success of RE in disciplining macroeconomic modeling should not obscure the fact that the term "rational" is merely a label. Rationality of actions and rationality of beliefs have little to do with each other and using the term "rational" in RE has tended to brand other beliefs as "irrational." Studying axioms of belief rationality is actually a fruitful area of research that can fill the wide open space between RE and true irrational beliefs.

A second point relates to private information. Not wishing to contradict RE, many adopted the device of asymmetric private information as the "cause" of diverse beliefs. Indeed, some view diverse beliefs as *equivalent* to asymmetric information. This is theoretically and empirically the wrong solution and in Kurz (2008, 2009) I explain why. It suffices to say that in large markets behavior under private information is different from behavior under diverse beliefs with common information. Also, all empirical evidence associates diverse forecasts with diverse modeling or diverse interpretation of public information. Finally, due to stochastic independence of private information over agents and time, RE models with asymmetric private information fail to deliver the key dynamic properties that are the central implications of

I thank George Evans, Carsten Nielsen, Volker Wieland and Mike Wolters for constructive comments on earlier drafts of this Introduction.

M. Kurz (✉)
Department of Economics, Stanford University, Stanford, CA 94305, USA
e-mail: mordecai@stanford.edu

Published online: 17 August 2010

economies with rational but diverse beliefs with common information (for details see Kurz (2009)).[1] Such dynamic properties entail added economic volatility and imply new risk that is generated *within the economy* and not by exogenous shocks. I discuss these dynamic properties below.

I cover three issues. First, I review reasons for belief diversity used in the papers. Next, I review the key substantive results of the papers. Third, although there is commonality of approaches among the papers, there are also subtle differences and a clarification of these ideas may then be helpful.

1 Why are beliefs diverse?

Belief diversity is not only a well-established empirical fact, but forecast data also reveal no tendency for persistent merging of opinions about any important variable. Hence, a good theory must explain not only belief diversity but also *persistence* of diversity. All papers in this volume hold views on this question but only two address it as a central research topic. Guesnerie and Jara-Moroni (2011) study the relation between the "eductive" viewpoint of expectational stability and rationalizability in games. Eductive reasoning has been used in static or in dynamic environments. To briefly explain this approach I note it emanates from questioning whether rational agents can coordinate their beliefs on a common belief that leads to stable dynamic path by assuming it is common knowledge all agents are rational and know the structure (e.g. Guesnerie 1992, 2002; Evans and Guesnerie 2003, 2005). To that end think of the economy as defining a reduced form dynamic iterative process in which a given set of d*iverse* agents' models induce a set of *true* possible transitions for the economy and each new true equilibrium transition leads to revised subjective models. For example, in a static Muth model agents can be asked to hypothetically choose a price forecasting model and this selection would lead to an equilibrium price given such functions. Given that equilibrium price functions are common knowledge, agents would revise their price forecast models which induce a true transition to a new equilibrium. Since under eductive reasoning the structure is common knowledge, rational agents are required to engage in such mental process even in more complicated dynamic settings. For belief coordination to occur this mental process of selecting forecast functions (or beliefs) must converge to a unique equilibrium which is the common belief. The process is close in spirit to Iterative Expectation Stability (e.g. Evans and Honkapohja 2001). Either lack of stability of the eductive process or multiple equilibria imply lack of coordination and hence persistence of diversity. If we think of forecast models as "strategies" then the process can be formulated as a game and conditions for self-fulfilling belief are analogous to conditions for rationalizability in games. In that case results in game theory become applicable to the study of diversity of beliefs. This is exactly the approach adopted by Guesnerie and Jara-Moroni (2011).

[1] In a static model REE with private information may not exist, may not be Pareto optimal, may not be incentive compatible, and may not be implementable as a perfect Bayesian equilibrium of an extensive form game. Since to ensure delivery under asymmetric information agents must write contracts, we find that a resulting REE fails to have desirable normative properties even if it exists [see Glycopantis et al. (2005, 2009)].

They examine different axioms of knowledge and rationality of agents in relation to conditions of rationalizability and show that even under strong conditions of knowledge and rationality, belief diversity can persist and convergence cannot be assured.

Wieland and Wolters (2011) explore the prevalence of belief diversity across macroeconomic models of the U.S. economy. They select six standard models and use the same historical data to estimate the model parameters and generate forecasts of output and inflation during the last five NBER-dated U.S. recessions. These constitute a collection of heterogenous forecasts which are then compared with the distribution of private forecasts by the Survey of Professional Forecasters (SPF). Such comparability is appropriate since data used are the same as data used by the Fed and SPF forecasters. The authors find that mean forecasts are close to the mean SPF and Fed's Greenbook forecasts. The degrees of heterogeneity among model and among professional forecasts are similar but these degrees vary substantially over time. Although all models assume agents hold RE, the diversity of model forecasts are traced to differences in assumptions, information sets, and estimated parameters across models. Hence, even a collection of standard academic models is a set of diverse beliefs.

The Wieland and Wolters (2011) paper has a powerful message with two parts. It shows that even within a traditional RE methodology there is a wide range of insufficient scientific knowledge which induces diverse beliefs since belief diversity always arises when some knowledge is absent. But since belief diversity causes volatility, the paper also shows that public policy cannot ignore the diversity among businessmen and among policy makers themselves. In rejecting RE, policy makers can thus take an important step towards improving the performance of public policy.

In explaining belief diversity the other papers fall into two groups that use different methods and assumptions and to conserve space I label them. The first group consists of Kurz and Motolese (2011), Guo et al. (2011), and Nielsen (2011). These adopt the Rational Belief (RB) approach due to Kurz (1994, 1997) and assume agents are rational but hold diverse beliefs due to their incomplete knowledge about the complex economy. The true dynamics is assumed non-stationary due to time-varying technology and institutions in which the structure changes faster than can be learned with precision from data. Since agents have past data on observables, the empirical distribution of exogenous variables is common knowledge. A model is said to be a RB if its simulation reproduces the empirical distribution. Diversity of belief is then possible since many different models are compatible with a given empirical evidence. In short, lack of knowledge of the truth is the foundation of belief diversity. Papers in this group specify beliefs about *exogenous* variables at the micro foundation level and then deduce forecast functions about *endogenous* variables from an equilibrium analysis. This leads to an important methodological development I discuss later, which requires agents to forecast the belief of others. Henceforth I refer to these papers as belonging to the "RB group."

The group consisting of Branch and Evans (2011), De Grauwe (2011) and Branch and McGough (2011) also assumes lack of full knowledge of the truth, but, in addition, they postulate agents have cognitive limitations that lead to boundedly rational forecast rules of endogenous variables. They assume forecast rules are chosen optimally from a restricted class of allowable misspecified rules. Central to this group is the fact that, following Evans and Honkapohja (2001), these are forecast rules of endogenous variables and hence are employed at the level of demand and supply rather than deduced from the micro foundations. Due to bounded rational forecasts, I henceforth refer to these three papers as the "BRF group." There is, however, a difference between Branch and McGough (2011) and the approach of Branch and Evans (2011) and De Grauwe (2011).

Branch and McGough (2011) study a model with two agent types: those using RE and those choosing from a fixed set of boundedly rational forecast models. Since boundedly rational agents use the same method of selecting models from the given set and since they all use the same data, all boundedly rational agents end up using the same forecast rule. The resulting diversity across groups is all the authors need for their results. In contrast, all agents in Branch and Evans (2011) and De Grauwe (2011) are boundedly rational. In equilibrium they are randomly assigned to forecast models in proportions (i.e. probabilities) defined by a multinomial logit distribution of the mean square errors (MSE) of past performance of the different models. Such selection may result in all agents selecting one model, but conditions are given to ensure equilibria exhibit belief diversity. The computed proportions measure the market diversity of beliefs. In Branch and Evans (2011) each forecast rule depends on parameters that are updated using least-squares, following Evans and Honkapohja (2001) and Branch and Evans (2006, 2007). These parameters and the forecast model proportions are endogenously determined in a manner I will explain later. In De Grauwe (2011) each forecast rule is based on a simple heuristic and the diversity proportions are just computed from past data. Although these are models of bounded rationality, they do entail partial rationality in ranking models in accord with their MSE and I explain later that this rationality is central to the dynamics of the economy.

2 The impact of diverse beliefs: a brief summary of results in this volume

I have already reviewed the main results of Guesnerie and Jara-Moroni (2011) and of Wieland and Wolters (2011). I focus now on the other six papers which fall into two categories. Kurz and Motolese (2011), and Guo et al. (2011) study *financial markets* while the other four study the effect of diverse beliefs on the conduct and efficacy of *public stabilization policy*.

Kurz and Motolese (2011) study the effect of market belief on risk premia. The dynamics of individual beliefs are described as Markov state variables, and the paper derives equilibrium asset prices and implied risk premium of a model where individual agents hold Markov beliefs. The paper shows that above and beyond the effect of business cycles on risk premia, fluctuations in market belief have significant independent effect on the time variability of risk premia. The analytical model leads to the conclusion that when the market holds abnormally favorable belief about future payoff of an asset, the market views the long position as less risky and hence the risk premium on that asset declines. This conclusion is tested empirically in markets for Federal Funds Futures, for 3- and for 6-month Treasury Bills. The empirical results are compatible with the theoretical prediction of the model.

Guo et al. (2011) study the problem of leverage with a three-period model of asset trading with leverage. In period 1 agents trade a risky asset using leverage where the risky asset pays either a High or Low dividend. They hold diverse probabilities of the High payoff which are RB in the sense of Kurz (1994). The distribution of their forecasts is specified to be wide and impacts the speculation level and use of leverage. The allowed leverage ratio is fixed exogenously. In period 2 an exogenous shock alters the belief of each agent with an Up or Down change. Given their new beliefs agents trade to adjust their portfolios. Optimistic traders increase their demand and pessimistic traders decrease it resulting in capital gains or losses. But then, in period 2 agents with capital gains can increase risk taking via leverage and those with capital losses may be forced to sell due to credit constraints. In period 3 assets pay dividend. The paper shows an increase in leverage ratio can raise demand of optimistic investors, result in a price increase at that date and in a "pyramiding" or "depyramiding" phenomenon of stock prices in the subsequent period. The paper also shows that an increase in the leverage ratio can, under some conditions, result in an increased price volatility.

Branch and Evans (2011) study monetary policy with a New Keynesian macro model of growth and inflation in which, as explained, agents forecast using misspecified models. Since the proportion of agents using each forecasting rule fluctuates in a manner I discuss later, the model generates internal volatility in output and inflation. Since there are only two possible forecasting classes, switching by a fraction of agents at any date has an externality effect which interacts with the policy rule. The authors use the term "misspecification equilibrium" to refer to the outcome. With this dynamics in place the model leads to several results. First, even if the central bank uses a Taylor-type monetary rule, multiple misspecification equilibria can emerge. Second, conditions are given to ensure that, in equilibrium, diversity of beliefs persists in the long run as time goes to infinity. Third, specific sequences of shocks can lead it to switch between an equilibrium with high volatility of output and inflation to one with low volatility. The paper then explores the implications for optimal discretionary monetary policy.

De Grauwe (2011) model is similar to Branch and Evans (2011) and I explore the precise differences later. He studies fluctuations in equilibrium allocations caused by fluctuations in market expectations which, in turn, are induced by a random choice between two allowable biased heuristic forecast. A De Grauwe (2011) equilibrium leads to sustained waves of optimism associated with high level of economic activity followed by waves of pessimism associated with low level of economic activity, each of which is of a long duration. Although the heuristic rules are simple, such simplicity allows the reader to gain a clear intuition of the process under study.

Branch and McGough (2011) study RBC model with a class of RE agents and a second class of boundedly rational agents whose decision rules are derived from a log-linear Euler equation at finite horizons. Their forecasting model is restricted to be a simple univariate process. Calibrated versions of the model with both types of agents are shown to imply impulse-responses to a standard technology shock that are hump-shaped, with output building up over time due to a greater increase in savings by bounded rational households relative to rational households. Due to amplification of volatility internally by misspecified forecast rules, the model can derive output volatility of empirical magnitude from smaller technology shocks than the standard RBC model. The propagation effect increases with forecasting/decision horizon and with the share of bounded rational households.

Nielsen (2011) asks whether a stabilization policy can be Pareto improving in the context of Rational Beliefs. This can be contrasted with earlier work (see Nielsen 2003, 2010) where ex-post optimality was used as a criterion for assessing policies which aim to restrict the impact of market belief on the economy. Compelling examples of volatile Pareto optimal allocations motivate the idea that society can be "better off" with reallocations of effective stabilization policies and this leads to the study of volatility reducing policies. Nielsen (2011) uses a three-period monetary OLG model with beliefs about final endowments which are diverse only across generations. In period 2 agents receive a non-random endowment and a random assignment of their belief types. Period 1 serves as a device to potentially allow an agent to ex-ante hedge his uncertainty about future types. However, markets are incomplete and such hedging is not feasible. In period 3 agents are passive and receive the random endowments. Since endowment in period 2 is not random, price volatility (which is actually the volatility of nominal interest rates) is caused only by fluctuations in market belief. The paper shows that a competitive monetary equilibrium is not Pareto Optimal, but under a specified set of assumptions the removal of volatility due to fluctuating beliefs leads to an allocation which is Pareto improving. It is then shown how a combination of monetary and fiscal policies can remove price volatility by neutralizing the effect of beliefs. The same problem is also studied in a two-type economy. Here again, it is shown that volatility reducing policies which lead to Pareto improving allocations exist.

3 On modeling the impact of belief diversity: an analytical guide

I now highlight the main modeling strategies employed in this volume. My aim is to help the reader see similarities and differences in methods used and understand why their authors chose those methods. Let me then start with terminology to simplify the discussion.

A belief is a probability on space of infinite sequences. It implies a sequence of date t joint *conditional* distributions on future events given date t information. "*Date t belief*" is a terminology I use to refer to date t subjective joint conditional distributions. In a Markov economy date t belief is a transition function. While the RB group defines date t beliefs via transition functions over *exogenous shocks*, the BRF group defines date t beliefs via forecasting functions of *endogenous variables*. Agents who believe an economy is stationary use a constant transition function or a fixed forecast function, and their forecasts change only if observed states change. Since dynamics revolves in this volume around changes in date t beliefs, I now turn to explore the manner in which these changes are described.

3.1 Describing individual and market states of belief: the expanded state space

I later examine why date t belief fluctuates. Here, I focus on the *description* of such changes and the concept used for that is the *individual state of belief*. This new state variable is common to all papers and is a variable which fully pins down an agent's date t belief. Since in most papers an agent's date $t+1$ belief is unknown at date t, *it* *expands an agent's state space of random events*. This methodological development is crucial. Denote by g_t^i agent i's belief state, then the vector $(g_t^1, g_t^2, \ldots, g_t^N)$ is *the market state of belief*. In a large economy the market belief—as a vector—is difficult to work with; hence it is natural to seek symmetry conditions of agents to permit aggregation. If it is possible, one defines market belief in terms of the moments of the distribution. It is common to study the first two moments—the mean and degree of diversity—defined by

$$\text{The mean market belief } = Z_t = \left(\frac{1}{N}\right)\sum_{i=1}^{N} g_t^i$$

$$\text{Cross-sectional standard deviation } \sigma_{Z^t} = \sqrt{\left(\frac{1}{N}\right)\sum_{i=1}^{N}\left[g_t^i - Z_t\right]^2}$$

Description of individual and market states of belief in papers by the RB group and by the BRF group are different and the reader may find it useful to detect such differences. Consider two examples. Kurz and Motolese (2011) introduce an exogenous dividend $\{\mathrm{d}_t, t = 1, 2, \ldots\}$ process for which the long-run empirical distribution is known to be characterized by the stationary transition

$$\mathrm{d}_{t+1} = \lambda_{\mathrm{d}}\mathrm{d}_t + \rho_{t+1}^{\mathrm{d}}, \qquad \rho_{t+1}^{\mathrm{d}} \sim N\left(0, \sigma_{\mathrm{d}}^2\right) \tag{1a}$$

This stationary transition is an average over a sequence of unobserved economic regimes due to technology and institutional changes. Agent i does not believe (1a) is the true date t transition and uses g_t^i to describe his own date t belief about d_{t+1} with the following perceived transition:

$$\mathrm{d}_{t+1}^i = \lambda_{\mathrm{d}}\,\mathrm{d}_t + \lambda_{\mathrm{d}}^{\mathrm{g}}\,g_t^i + \rho_{t+1}^{id}, \qquad \rho_{t+1}^{id} \sim N\left(0, \hat{\sigma}_{\mathrm{d}}^2\right) \tag{1b}$$

Not all RB papers follow the Markov transition of individual belief states. In Guo et al. (2011) individual belief states are specified as a linear cross sectional distribution at date 0 which is then subject to a random shock in date 1. Nielsen (2011) state of belief is an i.i.d. personal signal agents receive in the second date which specifies the distribution of their endowment in the third period. Also, in Kurz and Motolese (2011) market belief is represented by moments of the distribution $\left(g_t^1,\ g_t^2, \ldots, g_t^N\right)$ but in Guo et al. (2011) and in Nielsen (2011) market belief is the entire distribution.

As a second example consider Branch and Evans (2011). They take, as a starting point, an aggregate New Keynesian model of inflation π_t and GDP growth x_t of the form

$$x_t = \hat{\mathrm{E}}x_{t+1} - \varsigma\left(\mathrm{i}_t - \hat{\mathrm{E}}\pi_{t+1}\right) + \mathrm{v}_t \tag{2a}$$

$$\pi_t = \beta\,\hat{\mathrm{E}}\pi_{t+1} + \kappa\,x_t + \mathrm{u}_t \tag{2b}$$

where i_t is market interest rate set by monetary policy and $(\mathrm{v}_t, \mathrm{u}_t)$ is an AR(1) process. Agents forecast $y_{t+k} = (\pi_{t+k}, x_{t+k})$ for $k = 0, 1$ with a misspecified model of either $y_{t+k} = \mathrm{b}^1\, \mathrm{v}_{t+k} + \eta_{t+k}$ or $y_{t+k} = \mathrm{b}^2\, \mathrm{u}_{t+k} + \eta_{t+k}$ where η_t is a vector of random terms. These two choices are fixed exogenously, reflecting bounded rationality. The parameters $(\mathrm{b}^1, \mathrm{b}^2)$ are estimated via least squares given the incorrectly specified models, and past data is used to compute the MSE of the models, denoted by $(\mathrm{EU}^1, \mathrm{EU}^2)$. Following Brock and Hommes (1997) the model selection by agents is usually done stochastically and independently with proportions (or probabilities) determined by a multinomial logit distribution

$$n_1 = \frac{\exp(\alpha\, \mathrm{EU}^1)}{\exp(\alpha\, \mathrm{EU}^1) + \exp(\alpha\, \mathrm{EU}^2)}, \qquad n_2 = \frac{\exp(\alpha\, \mathrm{EU}^2)}{\exp(\alpha\, \mathrm{EU}^1) + \exp(\alpha\, \mathrm{EU}^2)} \tag{3}$$

Agent i's state of belief is then a random draw $g_t^i \in \{1, 2\}$ with $\mathrm{Prob}\left[g_t^i = 1\right] = n_1$ and $n_1 + n_2 = 1$. Individual draws are independent and agents are symmetric; hence the market belief is simply the pair (n_1, n_2) which describe the proportions of the g_t^i in the population. Diversity of beliefs amounts to the two classes of allowed beliefs. Within each class agents are identical but across classes their beliefs vary. Date t beliefs of agents vary over time due to the random selection of their belief at each date. If a very long data set is used to estimate $(\mathrm{b}^1, \mathrm{b}^2)$ and $(\mathrm{EU}^1, \mathrm{EU}^2)$, then both $(\mathrm{b}^1, \mathrm{b}^2)$ and (n_1, n_2) usually converge over time to constants. I later discuss how these may vary over time.

The above description also applies to De Grauwe (2011), but not to Branch and McGough (2011). These last authors set up a log-linearized RBC model where agents forecast their own future consumption, capital stock, and rate of return on capital. As explained, an agent either holds RE or is boundedly rational and selects from an allowable set of forecast functions, the parameters of which are estimated with least squares. Hence, the parameters of all boundedly rational agents are the same and all use the same forecast function. Individual states of belief are assignments each agent receives of which group he belongs to and this assignment is constant. The market state of belief is defined by the proportions of agents in the two classes and these proportions are also constant.

The distinction between individual and market beliefs has important implications. All papers postulate a large number of "small," anonymous agents each assuming his belief does not affect market belief. Moreover, all papers assume symmetry to ensure equilibrium endogenous variables depend only upon market belief and not on individual beliefs. So, assume an economy with a large number of agents holding beliefs that are randomly selected *independently.* Independence means that the law of large numbers renders average market belief constant and irrelevant to dynamics. Hence, belief *correlation* is needed for market belief to endogenously propagates volatility. Even in a simple noise trading model (e.g. DeLong et al. 1990) beliefs of all irrational traders are *perfectly* correlated and this condition is needed for them to affect equilibrium prices. Hence, to understand how volatility is affected by market belief the reader needs to detect the nature of correlation of beliefs within a model.

To broaden the discussion note the empirical facts that data on market belief exhibit *diversity and persistence*: when agents form beliefs they typically change them slowly. Persistence has two implications that need to be explored. First, as a challenge to RBC theory, persistence of beliefs imply fluctuations can be explained partly by persistence of technology shocks and partly by persistence of market belief. This is a key message of Kurz et al. (2005b), De Grauwe (2011) and Branch and McGough (2011).

Second, if market belief exhibits persistence then future market belief is predictable by today's market belief. The RB group of papers show that future belief diversity requires that an agent distinguish between uncertainty of his own belief tomorrow and tomorrow's market belief which is the belief of "others." Tomorrow's "belief of others" is a new state variable and random values of this state variable is a new kind of uncertainty which is different from the standard uncertainty about exogenous shocks. This leads to a further expansion of agents' state spaces. Do you learn anything new from observing today's market belief about tomorrow's *exogenous* shocks? Yes, but not about probability of future exogenous shocks. Why? Since market belief is based on the same information you already have, there is no new information (about future exogenous shocks) which you do not have and you can deduce from observing today's market belief! It is merely a collective opinion of "others" in the market about tomorrow's shocks. Since future market belief impacts tomorrow's prices, to predict prices tomorrow you need to forecast the belief of "others" tomorrow. Hence, belief persistence implies that today's market belief contains useful information which one uses to forecast tomorrow's prices! This explains why today's market belief contributes to the predictability of future asset prices.

It can be helpful to differentiate the method of the RB and the BRF papers. The achievement of the RB literature (e.g. Kurz 1974, 1994, 1997, 2008, 2009; Kurz and Beltratti 1997; Kurz and Motolese 2001, 2011; Kurz et al. 2005a,b; Kurz and Wu 1996; Motolese 2001, 2003; Nielsen 1996, 2003, 2010, 2011; Wu and Guo 2003, 2004; Guo et al. 2011) is the development of a consistent General Equilibrium theory with diverse beliefs where individual beliefs about exogenous shocks (hence market belief) have independent dynamics, on par with the dynamics of"fundamental" exogenous shocks. This literature assumes the economy is a non-stationary process and it is this complexity that leads to belief diversity. The impact of diversity is then the emergence of individual and market beliefs as two new state variables. *These expand the individual state spaces over which an agent's belief is defined* while preserving the unique nature of an agent's state space. This approach can lead to complex analysis but is useful to study the structure of equilibria while preserving the micro foundation of an agent's decision making.

In contrast, agents in models of the BRF group forecast endogenous variables ignoring the equilibrium relation between future exogenous and endogenous variables; hence there is no need for them to forecast explicitly the belief of others. The achievement of the BRF approach (e.g. Evans 1985; Evans and Honkapohja 2001, 2003; Branch and Evans 2006, 2007, 2010, 2011; Branch and McGough 2009; Branch and McGough 2011; De Grauwe 2011) is a drastic simplicity in describing the economy's dynamics, permitting transparent study of the macro implications of diverse beliefs. Dynamic stability and the impact of monetary policy are studied with relatively simple structures with clear results. One notes, however, that within the BRF group of papers there are differences in the dynamics of the market states of belief (i.e. (n_1, n_2) in the example). This issue is important and I return to it in discussing Amplification Mechanisms in the next section.

It may be useful to point out that papers in the BRF literature typically assume the true economic world of exogenous shocks is stationary and when equilibrium is stable and unique there is only one true forecast function. Complexity or time variability of the structure in the BRF literature arises from the agents' *learning process itself* and switching among forecast functions may reflect either multiple equilibria or boundedly rational process of searching for superior short-term performance.

In sum, a reader may benefit from focusing on the following issues: What is the correlation among individual beliefs? What causes market beliefs to fluctuate in a persistent manner? How does belief persistence affects the performance of the economy? These are issues I discuss now. The key insight is that *belief rationality is the principal cause of fluctuating beliefs and belief dynamics!*

3.2 Amplification mechanisms: rationality implies dynamics!

Consider an endowment economy with two agent types and random Markov endowments taking two values; hence the economy has two states. Under RE the economy has two prices. Suppose the two agent types have perfectly correlated "high" or "low" expectations of future endowments. Such fluctuating belief adds two more states; hence an equilibrium has four states and four prices. If forecasts are correlated *within* a type but not across types so a type can have high expectations while the other low expectations, the number of states rises to eight. Indeed, as diversity of partly correlated beliefs increases, the market exhibits more states and prices. Price proliferation effect is found in noise trading models, learning models, and in RB models. But price proliferation does not imply volatility! To express the idea of volatility due to market belief I proposed (Kurz 1974) the term *Endogenous Uncertainty*. Kurz and Wu (1996) define Endogenous Uncertainty to be present if prices are *different* in two states with the *same* exogenous shock. Some writers associate fluctuating beliefs with Keynes' Animal Spirits (see De Grauwe 2011). But then, why should date t beliefs fluctuate? I argue now that agents exhibit fluctuating beliefs *as a consequences of some rationality of belief!*

Irrational behavior can explain anything and hence it can "explain" fluctuating beliefs, which is how noise trading models work (e.g. DeLong et al. 1990) although there is nothing in the model to justify changes in date t beliefs.. It is, however, important to note that boundedly rational forecasting can cause amplification without changing beliefs. To dispose of this question I discuss first Branch and McGough (2011) where volatility is not amplified by fluctuating beliefs. Their forecasters select one of two models and estimate its parameters using past data. Such forecast models amplify fluctuations relative to standard RBC models. But what causes this amplification? To provide intuition note that decision rules are deduced from forecasts. If you forecast no change your decisions are likely to change only little. But if you forecast drastic changes your decisions are likely to change drastically. Hence, introduction of sufficiently misspecified forecast models can introduce sufficient variability into actual decisions so as to amplify economic fluctuations without fluctuating date t beliefs. This is what Branch and McGough (2011) demonstrate and in doing so they complement Kurz et al. (2005b) and De Grauwe (2011) results on the amplification of business cycles with fluctuating beliefs (see below).

Returning to irrational noise trading, observe it is equally compatible with constant date t belief. Irrational agents may always be optimistic or always be pessimistic, disregarding evidence. The terms "optimism" or "pessimism" have meaning only in relation to a reference empirical evidence. Papers in the RB group assume agents do not start at date 0 and have ample past data to compute the empirical distribution of past observed *exogenous* variables. They compute relative frequencies of events from which they deduce a stationary probability which is their reference empirical evidence. In a Markov world they deduce a stationary transition like (1a) taken from Kurz and Motolese (2011). Optimism or pessimism are date t beliefs which assign higher or lower probability than the empirical probability. Although there is no generally accepted definition of belief rationality, most agree rationality requires compatibility with evidence. I thus discuss the RB group of papers separately from the BRF group.

The RB principle implies that for (1b) to be a RB its empirical distribution must be (1) which depends upon the time variability of g_t^i in (1b). But as a minimum, any rationality axiom would require that forecasts are correct *on average*; hence the mean forecast error is zero. Such a minimal condition implies a belief cannot lead to a *constant transition* unless an agent believes the stationary transition (1) is the truth. This is so since if he holds a *constant* transition which is not (1), then his mean forecast is different from the mean forecast under (1); hence his forecast error does not have a zero mean. Simply stated, an agent who is always optimistic or always pessimistic relative to (1) is irrational since his belief is rejected by the data. It follows *that rationality implies date t beliefs of that agent must fluctuate*! Since all data show persistent belief diversity, most agents do not hold (1) as their belief; hence their date t beliefs must fluctuate. What are the causes of these fluctuations? Many reasons are given such as different models for learning from data, Bayesian updating with diverse priors when posterior beliefs do not converge, diverse interpretation of public information, etc. Finally, given the empirical evidence for *diverse and persistent* beliefs, one concludes that most agents hold fluctuating date t beliefs which are often imprecise or even wrong while being rational and being "wrong" are not in conflict. Rational agents hold wrong beliefs as long as there is no evidence against them and the term *"wrong"* is obviously used here relative to an unknowable standard.

Turning now to the BRF group I focus on Branch and Evans (2011) and De Grauwe (2011). Although their agents are boundedly rational, they are sufficiently rational so as to rank performance of the allowable class of models and select randomizing with the multinomial logit probability in (3). Since random draws of these beliefs are independent across agents, only the dynamic properties of market beliefs (n_1, n_2) matter. These fluctuate because agents rank forecast rules by using only recent data (i.e. regressions with constant or non-decreasing gain) to estimate recent success of different models. Large shocks or specific sequences of random shocks in the data can trigger a sustained switch among multiple equilibria, implying shifts from one market distribution (n_1, n_2) to another. In particular, it is possible for a "bad luck" event to shift the economy from a low- to a high-volatility regime.

Within the BRF group there is a subtle difference in the definition of equilibrium. Motivated by stability conditions of Evans and Honkapohja (2001) and by rationalizability as in Guesnerie and Jara-Moroni (2011); Branch and Evans (2011) require, as a definition of equilibrium, that market belief proportions (n_1, n_2) map into themselves. I note that in Guesnerie and Jara-Moroni (2011) such a fixed point property arises from a conceptual experiment in each agent's mind, based on a common knowledge of the structure. In Branch and Evans (2011) the fixed point is motivated by the idea that market beliefs should be rational in being *self-confirming*, and implicit in the definition is a coordination game where agents select beliefs and can deviate from a belief "consensus," rendering a non-fixed point proportions unstable. Such strict equilibrium requirement leads to the possibility of multiple equilibria. I note that with learning under decreasing gain such fixed point stability often implies convergence to equilibrium with constant market belief (which may, however, include diverse beliefs). Fluctuations of the aggregate proportions thus depend in an *essential* way on learning only from recent data and the implied fluctuations may entail switching among multiple equilibria as defined above.

De Grauwe (2011) definition of equilibrium does not require the fixed point property explained above. At each date agents compute MSE of the allowable forecast models, inducing a random assignment of agents to new forecast rules as in (3). Once forecasts and decisions are made the economy proceeds into the next date when new MSE are computed. Market belief (n_1, n_2) changes slowly with new data and agents recompute the implied new MSE. Since (n_1, n_2) do not satisfy the fixed point condition explained above, fluctuations in market belief do not entail switching among multiple equilibria but rather, these are dynamic fluctuations within a given equilibrium. Although both study dynamics under monetary policy, one can see that the Branch and Evans (2011) dynamics is somewhat different from De Grauwe (2011).

3.3 Correlation as belief externality

I have argued earlier that belief rationality is an important component of a coherent theory of belief diversity since rationality is at the heart of its dynamic implications. One must then expect that empirical tests of belief rationality relate to the implied dynamics in addition to orthogonality. However, I must now qualify this position and show there are important belief externalities which cannot be deduced from rationality principles; hence a theory has to leave room for exuberance and collective irrationality. It is found in the structure of correlation across individual beliefs.

To explain the role of correlation I continue to use, as an example, the belief dynamics of Kurz and Motolese (2011). They prove that the state variable g_t^i has a transition of the form

$$g_{t+1}^i = \lambda_Z\, g_t^i + \lambda_Z^{\mathrm{d}}\, \mathrm{d}_t + \rho_{t+1}^{ig}, \qquad \rho_{t+1}^{ig} \sim N(0, \sigma_{\mathrm{g}}^2) \tag{4}$$

The random term is crucial and they show it arises from a random arrival of qualitative information. The mean market belief (i.e. the mean of the g_t^i) is denoted by Z_t. Hence, averaging (4) leads to

$$Z_{t+1} = \lambda_Z Z_t + \lambda_Z^{\mathrm{d}} \mathrm{d}_t + \frac{1}{N} \sum_{i=1}^{N} \rho_{t+1}^{ig}$$

Since individual beliefs are correlated the law of large numbers does not apply and the average of the ρ_t^{ig} over *i does not vanish*. Defining $(1/N) \sum_{i=1}^{N} \rho_t^{ig} = \hat{\rho}_{t+1}^{Z} \neq 0$ one concludes that

$$Z_{t+1} = \lambda_Z Z_t + \lambda_Z^{\mathrm{d}} \mathrm{d}_t + \hat{\rho}_{t+1}^{Z} \tag{5}$$

The term $\hat{\rho}_t^Z$ is crucial to dynamic fluctuations in the mean, but $\hat{\rho}_t^Z$ *is not subject to rationality*. It arises from social interaction, it can result from herding or from other forms of collective exuberance. For example, suppose the correlation between ρ_t^{ig} and ρ_t^{jg} is R_t which is time varying correlation, independently of i and j and, as explained, is arbitrary; hence the random terms in (4) satisfy $\mathrm{Cov}(\rho_t^{ig}, \rho_t^{\mathrm{jg}}) = R_t \sigma_{\mathrm{g}}^2$. In that case one shows $\mathrm{Var}(\hat{\rho}_t^Z) = \sigma_{\mathrm{g}}^2 R_t$ for large N. Each individual random term ρ_t^{ig} has a *constant* variance while the *mean exhibits time varying volatility*! Moreover, if ρ_t^{ig} exhibits small serial correlation, the time path of $\hat{\rho}_t^Z$ has a wide degree of freedom. This fact explains my own view that the most important cause of volatility in financial markets is the structure of correlation across traders and the shifting nature of this correlation. Since market belief impacts macro performance, a crucial component of the theory is yet to be developed not from the perspective of individual choice (rational or irrational) but from the perspective of the functioning of social institutions. The above example of correlation is representative of other papers of the RB group.

Correlation of beliefs is also central among the BRF group. In assuming agents select from two forecast rules Branch and Evans (2011) and De Grauwe (2011) ensure agents' forecasts are correlated. After random assignment, forecasts are perfectly correlated within each of the two groups. Moreover, correlation across agents and serial correlation of forecasts over time—as new MSE of forecast rules are computed—both play a crucial role in the dynamics of the economy. Although some correlation simply result from the *structure* of the model, the important fact is that it does not arise from individual rationality and is taken as given by individual agents. Once given, it operates as an externality.

The issue of correlation complicates the problem of testing for rationality. Since there would be no aggregate dynamics without correlation, such correlation can entail market herding, bubbles, and other forms of *collective* behavior. As aggregate phenomena they do not necessarily provide evidence for individual irrationality.

3.4 The centrality of heterogeneity: can a representative agent be sufficient?

Is belief heterogeneity essential to these papers? Can their results be deduced from models with a representative agent who does not hold RE? Although the two answers are "yes" and then "no," there are subtle differences in the reasons for these answers across the papers. I briefly outline them now.

(i) The RB approach implies that to forecast prices, an agent forecast the belief of "others," whose belief he perceives to be different from his own. This distinction does not exist in a representative agent economy in which uncertainty about the belief of others disappears.
(ii) Heterogeneity plays a key role in the dynamics of market belief via the structure of *correlation* across beliefs which impacts the implied dynamics. Hence, changes in heterogeneity without changes in the mean matter but are missed by a representative agent.
(iii) For the BRF group with multiple misspecified forecast models there are conditions when heterogeneity is not a long-term result. This, however, is not the general case.
(iv) Substantial literature shows that the cross-sectional standard deviation across individual forecasts contributes to the predictability of returns, and this is the empirical conclusion of Kurz and Motolese (2011) as well. Cross sectional variance does not exist in a representative agent economy.
(v) Can a belief heterogenous economy be replicated by a representative agent? There are several results in the literature, including Kurz and Motolese (2011), that show average market belief can be neither a probability nor rational; with diverse beliefs one cannot construct an "average" state space.

4 Concluding remarks

This volume offers a sample of papers which are part of a growing literature advancing the view that market belief is a central cause of economic volatility and consequently market risk is not just about exogenous shocks but mostly about the effects of future market expectations. Since this point of view raises questions which are fundamental to the functioning of an economy, it is thus natural the problems tackled by these papers represent a sample of the open questions for future research. To be more specific let me offer four examples.

(i) Wieland and Wolters (2011), Branch and Evans (2011) and De Grauwe (2011) point out the need for a reformulation of the basic tools of Macroeconomics to account for the effect of diverse beliefs and to study the impact of such a fact on the conduct of monetary policy.
(ii) Nielsen (2011) shows that with diverse beliefs and high volatility Pareto optimum is not compelling as a criterion to support stabilizing monetary and fiscal policy, and additional research is necessary to study optimal stabilizing policies taking into account *the need to neutralize the effect of market belief.*
(iii) Branch and McGough (2011), Branch and Evans (2011) and De Grauwe (2011) highlight the impact of expectations on business cycles and this remains an important area for future research.
(iv) Guo et al. (2011) stress the crucial interaction between credit constraints and market expectations as a mechanism impacting economic volatility. The role of credit creation is important and not sufficiently studied. This observation is particularly relevant to the financial crisis of 2008–2009 and complements my point (iii) about the role of expectations in business cycles. Events of recent

years demonstrate the strong combined effect of low credit restrictions and market expectations on economic volatility, business cycles, and the effect of policy. In this context the problems addressed by Guo et al. (2011) together with other papers in this volume offer a rich set of questions for future study.

References

Branch, W.A., Evans, G.W.: Intrinsic heterogeneity in expectation formation. J Econ Theory **127**, 264–295 (2006)

Branch, W.A., Evans, G.W.: Model uncertainty and endogenous volatility. Rev Econ Dyn **10**, 207–237 (2007)

Branch, W.A., Evans, G.W.: Asset returns and the dynamics of learning. Rev Financ Stud (2010, forthcoming)

Branch, W.A., Evans, G.W.: Monetary policy and heterogeneous expectations. Econ Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0539-9

Branch, W.A., McGough, B.: A new Keynesian model with heterogenous expectations. J Econ Dyn Control **33**, 1036–1051 (2009)

Branch, W.A., McGough, B.: Business cycle amplification with heterogenous expectations. Econ Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0541-2

Brock, W.A., Hommes, C.H.: A rational route to randomness. Econometrica **65**, 1059–1095 (1997)

De Grauwe, P.: Animal spirits and monetary policy. Econ Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0543-0

DeLong, B., Shleifer, A., Summers, L., Waldman, R.J.: Noise trader risk in financial markets. J Political Econ **98**, 703–738 (1990)

Evans, G.W.: Expectational stability and the multiple equilibria problem in linear rational expectations models. Q J Econ **100**, 1217–1233 (1985)

Evans, G.W., Guesnerie, R.: Coordination on saddle-path solutions, the eductive viewpoint: linear univariate models. Macroecon Dyn **7**, 42–62 (2003)

Evans, G.W., Guesnerie, R.: Coordination on saddle-path solutions, the eductive viewpoint: linear multivariate models. J Econ Theory **124**, 202–229 (2005)

Evans, G., Honkapohja, S.: Learning and Expectations in Macroeconomics. Princeton: Princeton University Press (2001)

Evans, G., Honkapohja, S.: Expectations and the stability problem for optimal monetary policies. Rev Econ Stud **70**, 807–824 (2003)

Glycopantis, D., Muir, A., Yannelis, N.C.: Non-implementation of rational expectations as a perfect Bayesin equilibrium. Econ Theory **26**, 765–791 (2005)

Glycopantis, D., Muir, A., Yannelis, N.C.: On nonrevealing rational expectations equilibrium. Econ Theory **38**, 351–369 (2009)

Guesnerie, R.: An exploration of the eductive justification of the rational expectations hypothesis. Am Econ Rev **82**, 1254–1278 (1992)

Guesnerie, R.: Anchoring economic predictions in common knowledge. Econometrica **70**, 439–480 (2002)

Guesnerie, R., Jara-Moroni, P.: Expectational coordination in simple economic contexts, concepts and analysis with emphasis on strategic substitutabilities. Econ. Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0556-8

Guo, W.C., Wang, F.Y., Wu, H.W.: Financial leverage and market volatility with diverse beliefs. Econ. Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0548-8

Kurz, M.: The Kesten-Stigum model and the treatment of uncertainty in equilibrium theory. In: Balch, M.S., McFadden, D.L., Wu, S.Y. (eds.) Essays on Economic Behavior Under Uncertainty, pp. 389–399. Amsterdam: North-Holland (1974)

Kurz, M.: On the structure and diversity of rational beliefs. Econ. Theory **4**, 877–900 (1994). (An edited version appears as Chapter 2 of Kurz, M. (ed.) 1997)

Kurz, M. (ed.): Endogenous economic fluctuations: studies in the theory of rational belief. In: Studies in Economic Theory, No. 6. Berlin and New York: Springer-Verlag (1997)

Kurz, M.: Beauty contests under private information and diverse beliefs: how different? J Math Econ **44**, 762–784 (2008)

Kurz, M.: Rational diverse beliefs and economic volatility. In: Hens, T., Schenk-Hoppé, K.R. (eds.) Handbook of Financial Markets: Dynamics and Evolution, Chap. 8, pp. 439–506. Amsterdam: North-Holland (2009)

Kurz, M., Beltratti, A.: The Equity Premium is No Puzzle, Chap. 11, pp. 283–316. In: Kurz, M. (ed.) (1997)

Kurz, M., Motolese, M.: Endogenous uncertainty and market volatility. Econ Theory **17**, 497–544 (2001)

Kurz, M., Motolese, M.: Diverse beliefs and time variability of risk premia. Econ Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0550-1

Kurz, M., Wu, H.M.: Endogenous uncertainty in a general equilibrium model with price contingent contracts. Econ. Theory **8**, 461–488 (1996). (Published also as Chapter 2 of Kurz, M. (ed.) 1997)

Kurz, M., Jin, H., Motolese, M.: Determinants of Stock Market volatility and risk premia. Ann Finance **1**, 109–147 (2005a)

Kurz, M., Jin, H., Motolese, M.: The role of expectations in economic fluctuations and the efficacy of monetary policy. J Econ Dyn Control **29**, 2017–2065 (2005b)

Motolese, M.: Money non-neutrality in a rational belief equilibrium with financial assets. Econ Theory **18**, 97–126 (2001)

Motolese, M.: Endogenous uncertainty and the non-neutrality of money. Econ Theory **21**, 317–345 (2003)

Nielsen, C.K.: Rational belief structures and rational belief equilibria. Econ Theory **8**, 339–422 (1996)

Nielsen, C.K.: Floating exchange rates verrus a monetary union under rational beliefs: the role of endogenous uncertainty . Econ Theory **21**, 347–398 (2003)

Nielsen, C.K.: Optimal economic institutions under rational overconfidence with applications to the choice of exchange rate regime. Int J Econ Theory (2010, forthcoming)

Nielsen, C.K.: Price stabilizing, pareto improving policies. Econ. Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0555-9

Wieland, V., Wolters, M.: The diversity of forecasts from macroeconomic models of the U.S. economy. Econ. Theory **47** (2010, in this issue). doi:10.1007/s00199-010-0549-7

Wu, H.M., Guo, W.C.: Speculative trading with rational beliefs and endogenous uncertainty. Econ Theory **21**, 263 –292 (2003)

Wu, H.M., Guo, W.C.: Asset price volatility and trading volume with rational beliefs. Econ Theory **23**, 461–488 (2004)